**Mello GDMPK**

Mentions that you can use a max-entropy ansatz (equivalent to isotropy ansatz) to characterize the transfer matrix (at least the infinitesimal one?). And these results work well for Q1D. He says that in the polar decomposition:



the matrices u, and υ are independently distributed. They are random elements of U(N). This is equivalent to the isotropy assumption, and a consequence of the max-entropy assumption I believe. The isotropic model (IM) does not allow a depiction of diffusion in the transverse direction, since it is assumed that that diffusion has already taken place – which is why isotropy occurs. And he says that calculations performed in the IM don’t agree with other theoretical predictions/numerical simulations for 3D conductors with larger disorder I guess. OK, now let’s develop the equation which governs the evolution of quantities that depend on M. Consider a function F(M). And let M obey the recursion relation:



I would take each slice m(n) to be of thickness δx, which could be finite small thickness at this point. The recursion relation is:



and P(M(n)) is given by:



But <δ> is zero, and we will end up needing only the second order term and so we get:



At this point we should define our expectations. M is given by:



Next he expands the matrix products out to order δL. In terms of ε, then we have:



Now he writes α = eθ where θ = ih and h is a Hermitian matrix. This is certainly allowed. Let’s expand the first m-1 in powers of θ, β, which will be assumed to be small in some sense to be made more explicit shortly; basically θ, β are being taken ~ √δx. If we expand out to order δx, then we get:



He postulates (WSL)



So this amounts to:



Yeah, his expectations are a bit off from the δ-correlation model that Tart uses.



Also, another model we’ll use is the equivalent channel model:



I’m not sure how this would be obtained from my model. My δ and Born approximation models are of form σab = τaτb. Are these models incapable of descriping EC (equivalent channels)? How does the EC model develop from the NEC model? We have the matrix element σab(V2,L). Taking V2 → 0, and then L → 0 results in what I have – perhaps a ballistic model. I surmise that if we get 0 < V < Vc but still finite and then take L → 0 limit, we would get the equivalent channels model. This might happen because going out to higher order in V certainly changes the form of the expectation, as we get sums over products of C’s. And then perhaps if we have V > Vc with L → 0, we would get the insulating model? Or maybe there is no VC in our model because we neglect transitory modes? Well in any event, we can now say:



Filling this into our P(M), and then into our evolution equation we will eventually get:



Mabcd is understood as follows: ab refers to the blocks 4 block matrices that comprise M, and cd are their elements. And it follows that the probability distribution function for M is given by:



But it seems to me that the expansion should be in terms of the real/imaginary parts of M or the complex/conjugate parts. So he has left these out? What would these look like? Let’s separate into real/imaginary parts first. Going back to the beginning of the expansion, we have:



The last line follows from the penultimate because ε is a real operator. Now let’s do complex/conjugate parts:



Yeah, we can do the same thing (let s stand for complex, conjugate), because ε is real. Certainly not the prettiest equation. But this appears to be what one would get if you use the toy models I’ve been using. He asks, can you obtain this equation by using a maximum entropy ansatz for the slice, subject to constraints that the expectations of the slice work out as above? We should look at Shukla paper for ideas here. Would like to integrate out the u and υ matrices to get information about just λ, but that seems hopeless unless one can first solve for p itself. And that seems…complicated. So how would we turn this into a single parameter theory? We would have to model σab with a single parameter like we do Kab? Or perhaps, when solve for g(L), it will turn out to depend on only a single of these parameters?

He shows that flux is conserved by writing out the evolution equation for Q = MΣzM† and shows that ∂<Q>/∂L = 0. I think this was in doubt because the equation isn’t just in terms of polar matrices which would automatically imply flux conservation.

Now we talk about determining the evolution of the reflection and transmission amplitudes. We start with t´ab, where t´ can be given in terms of M via,



So t´ = M22-1. [just have to put t, r, etc., the restricted set of variables in terms of any combination of our more general set of variables] If we were to determine the evolution of this quantity, then we’d use the identity (via implicit differentiation),



So then,



Continuing,



where we have used the definition Σjσjb = σb, which is the m.f.p. of channel b. So then we’ve got:



and therefore,



Now for a time-reversed system, we have t’ = tT. So we can write:



with the obvious solution:



The isotropic model, in contrast gives <t> = 0 for all lengths. Well for instance, t-1 = uυ/√(1+λ), and

t = υ†u†√(1+λ). And I would expect that this ought to give 0 on average, since the u, υ are independently isotropically distributed – the average of a phase is zero in 1D at least. Note that we made this assumption, implicitly, in our formulation of the GDMPK. What if we tried to evaluate <t> from M11? Well that wouldn’t work because <M11> = 1 always. And cannot get <M-1> from <M> anyway. We can extend this to the moments of t. Consider <tabta´b´\*>. We can take derivatives of t\* using t† = M11-1 → t\*ab = (M11-1)ba. And then we will find:



Obviously this equation cannot be solved self-consistently unless we knew the reflection coefficients. If we set a=a, b=b´, then we have the transmission probability from b to a.



We can at least say that at L = 0, we should find all the t’s = δab, and r’s = 0 and so we can say that:



I wish we could demonstrate how this evolves into 1/N, independent of L for large L. Appropos the reflection coefficients, we can calculate the evolution of <r´ab>. We get:



with initial condition <r´ab> = 0, this yields the inevitable result, <r´ab> = 0 for all L. More interesting is the following average <r´abr´a´b´\*>. We can take derivatives of r\* because r = -t´M21 = -M22-1(-M21) = M22-1M21. And therefore r\* = M22-1\*M21\* = M11-1M12. I guess something similar can be worked out for r´.



So evolution of <r2> involves <r2> and <r4>. And we can expect that evolution of <r4> will involve <r4> and <r6>? In any event, this presages the fact that the averages <r2>, <r4>, <r6>, …., will together form a closed set, albeit with an infinite number of equations. And these could be solved in principle. The reflection probability is given by:



Evaluating at L = 0, we can say:



He works out the O(L2), via application of P twice of course. And he gets:



Now we turn our interest to the total transmission coefficient,



We will consider a perturbative calculation of R. So summing <|r´ab|2> over *a* and *b*. Then we get,



(recalling σ´a = σa). And he adds a third order term, to get:



So then we have to first order:



Specializing to the equivalent channels model, whereby σab = (1+δab)/(N+1)ℓ, etc., we get:



And of course T = N – R. Next lets consider the evolution of Tr(λ). To facilitate this, define y = uλu†. Note that y = M12M12† = M12M21T. Then of course Tr(λ) = Tr(y). Now let’s apply the evolution equation to yaa = M12amM21am = Σm|uam|2 λm. Note that this makes yaa the small slice reflection coefficient for this channel (on right side) for small λm. |uam|2 is the probability of current in channel *a* scattering into channel m, and λm is the approximate probability of reflection.



These M products are:



and so we have:



which we can write as:



And so we see that this is a closed set of linear differential equations for the trace elements of y, which we can solve. Let’s write it in matrix notation, and then we can write down the homogenous and particular solution:



Imposing initial condition we have:



So our answer is:



σ-1|σ> is actually simple – it’s just |1> = |111…111>, as you may verify. Yep. So our solution can be written



Now let’s diagonalize the matrix σ via σ|s> = (1/ℓs)|s>, where |s> is the eigenvector and 1/ℓs is the associated mean free path. Then we have:



So,



Well we actually want Σyaa, so doing this we find:



In the large L limit, only the smallest mean free path would matter and we’d get:



Well so this is the exact result using those expectations. And I wasn’t able to get it. In fact I even encountered divergences when I did it the simplified way, and appeared to encounter these even when I didn’t. I’ll have to look into this more closely later. So looking into it… Now let’s specialize to the product form σ, i.e. σab = τaτb. σ has a normalized eigenvector, |τ>/τ, where τ = √(τaτa). Its eigenvalue can be ascertained: σabτb/τ = τaτbτb/τ = τaτ and so the eigenvalue is just τ = 1/ℓs. The other eigenvectors are perpendicular to |τ>/τ (naturally) and have 0 eigenvalue. How know? Because if they are perpendicular to τ, then when do σabξb = τaτbξb = τa<τ|ξ> = 0. Also, τ is the largest eigenvalue and therefore minimum ℓ, as we were looking for. Now in δ model τa ~ 1/ka, where ka is the longitudinal momentum. So τ = Σa1/Aka2 = 1/AK2 (in my notation).

Anyway, if we specialize to the equivalent channel case, then we’ll have: σab = (1+δab)/(N+1)ℓ, and all the m.f.p.’s are the same and so we’ll get:



noting that <1|1> = N. We can obtain some information about the index dependence of the eigenvectors |uab|2. Consider the least transmitting channel, which we’ll call N. This is the one with the largest eigenvalue λN. Then we have:



(keeping only index dependent terms). Now let’s specialize to the product form σ, i.e. σab = τaτb. σ has a normalized eigenvector, |τ> = τa/τ, where τ = √(τaτa). Its eigenvalue can be ascertained: σabτb/τ = τaτbτb/τ = τaτ = τ2τa/τ and so the eigenvalue is just τ2. The other eigenvectors are perpendicular to |τ> (naturally) and have 0 eigenvalue. How know? Because if they are perpendicular to τa, then when do σabξb = τaτbξb = τaτ<τ|ξ> = 0. Also, τ2 is the largest eigenvalue and therefore minimum ℓ, as we were looking for. So then we have:



Now in δ model τa ~ 1/ka, where ka is the longitudinal momentum. So:



where N is the number of channels, and a is the index enumerating the channel wavevectors. As can see, the projection of the least transmitting channel (N) onto the channels (a) is largest for a ~ N, i.e., large values of kTa, which means small values of kaz. And it seems to be common sense that the least transmitting channel would be reflected onto itself most likely, and other channels, less likely. And this implies that large transverse momenta tunnel through the sample least effectively, which seems quite reasonable. In contrast, in the isotropic model, all channels tunnel through equally well. Could this formula be used to determine information about Kmn = Σa|υma|2|υna|2? Consider:



Deep in the insulating regime at least. And we may say that this is approximately,



well not quite. Now let’s consider the probability distribution of the eigenvalues λ. It is difficult to determine the evolution of F(λ1, λ2, …, λN). So instead of N λ’s, we’ll consider the time development of N generalized traces, F(ρ1, ρ2, …, ρN), where ρk is a generalized trace, defined below:



where as before y = uλu†. These could be considered the ‘moments’ of the small slice reflection probability. Note that y = M12M12† = M12M21T. Now let’s consider the evolution equation for some unspecified F(**ρ**). Now ρk depends on two quantities: M12 and M21. Let’s call these x and y. And therefore F(ρ1,ρ2,…,ρN) depends on **M** through these two quantities as well. So we can say: F(ρ1(M12, M21), ρ2(M12, M21), …, ρN(M12, M21)). We know of course that ∂<F>/∂L = <PδLF>. What we want to do, is somehow pass the PδL operator through the F and onto the individual ρ’s. So to start, the evolution of F would look like,



And now working out the derivatives, we have to evaluate the generic term, c(M,M‘)∂2/∂M∂M΄, with an implicit summation over all the indices of M and Mʹ, and c(M,Mʹ) being some arbitrary function of the two variables. Obviously, this sufficiently represents the operator P for our purposes. So,



Now we would like to write the first term as ∂2/∂M∂M΄ {something}. To that end, consider:



Inserting this into the first term in the ∂2F/∂M∂M´ expression, due to the symmetry of the term, we see that we can write:



And therefore we can say in general, since the evolution operator P is made up entirely of such derivatives, that:



Note this identity would hold even if P had a single derivative part as well: c(M)∂/∂M. If you plug in the more general P into the equation you’ll see it’s true. Note especially that the [ ] term will cancel out the single derivative part because its in the form of the product rule (difference). So we can say:



Let’s consider therefore the action of P on ρk and ρj. Well, I’ll quote his result. He gets, for EC:



Filling this into our evolution equation for F we get:



And he notes that the ρα for α > N (for note that j+k, and j+k-1 may exceed N), can be expressed uniquely in terms of ρj (j < N). Next, if we let F = δ(**ρ** – **ξ**), where **ρ** = (ρ1, ρ2, …, ρN), then we can develop an evolution equation for this quantity. So…



and so our evolution equation becomes:



And now he says that this same evolution equation for p(**ξ**) can be obtained using the DMPK equation (we would just integrate p(λ) against p(**ρ**) or something). And so this proves that we could derive the DMPK equation from the EC assumption. So the DMPK equation is really a test of equivalent channels, not isotropy per se´.

**How express higher traces in terms of lower ones?**

So,



So maybe you could solve for λ1 and λ2­ in terms of Tr(λ) and Tr(λ2), and then you could get any higher order trace by virtue of this. For instance,



Seems like this equation might not have solutions. Yeah might not. Well in any event, can at least use substitution method and get λ1,2.

**What exactly is <dMdM>Mʹ?**

So we have:



then,



He postulates (WSL)



Well, let’s do this (η = β)



So,



Now



So this effectively reduces to:



And since cross terms are zero, this reduces still more to:



which can be expressed as:



Note that this is the same form as the white noise model. And we have the same general expectations, except that the δ’s would be replaced by C’s. And the σ’s are identical. Let’s see what the equivalent channel model yields:



**Can you obtain p(λ) from p(M) in 1D?**

In general we have:



which in 1D reduces to:



which goes to:



Combining like terms we get:



Yuck.

**What does his statistical model predict for GDMPK?**

So his model is, again,



(θ is anti-Hermitian, and β is symmetric) He postulates (WSL)



So, following the Chalker calculation,



and,



and,



The perturbation, and matrix W, is:



Keeping only contributing combinations:



Now the components are:



and now let’s take expectations,



and for the next correlation,



and simplifying:



and in particular, in the diagonal approximation, we’d have:



and another one we need is:



Now let’s look at the eigenvalue expansion:



Well whatever. Let’s look at the previous expectations:



So these are the same. So Mello’s model reduces to the white noise model, at least so far as the eigenvalue distribution is concerned.